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THESIS

A METHOD TO CALCULATE THE PARAMETERS OF WINGS OF ARBITRARY PLANFORM

by

Edward Mark Barber

December 1984

Thesis Advisor: T.H. Gawain

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A Method to Calculate the Parameters of Wings of Arbitrary Planform

by

Edward Mark Barber
Lieutenant, United States Navy
B.S., University of New Mexico, 1977

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN AERONAUTICAL ENGINEERING

from the

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December 1984

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I. BACKGROUND

The overall research of which this thesis forms a part has a three-fold purpose. Firstly, it is intended to instruct the student about the fundemental equations that govern the aerodynamics of wings in incompressible flow. Secondly, it teaches the student how to convert the fundemental equations into a form suitable for numerical calculation on the digital computer and to carry out the complex programming required for this purpose. Thirdly, once the first two objectives have been accomplished, the computer program itself provides a useful pedagogical and design tool for illustrating the effects of various wing design parameters on the final aerodynamic performance of a wing.

Lcdr. J. L. Parks made a creditable start toward the first objective in his thesis [Ref. 2]. However, in the limited time available to him, he was unable to make significant progress toward the second or third objectives.

It was evident early that Parks' computational technique had some flaws. The expectation was that these would be readily discovered and soon corrected and that objectives two and three would be rather quickly and easily completed. It has not worked out that way!

As the investigation actually developed, a whole series of obstacles were encountered and each was in turn eventually overcome. But the process was neither quick nor easy. In every case, it was found that the seeming obstacle was really attributable to some conceptual error. Once the error was found, the obstacle disappeared and the investigator gained a deeper insight.

Among the obstacles that were encountered and eventually overcome were the following;

1) Confusion about the proper type of computational grid to use, whether staggered or unstaggered, how fine, and so on. We have now settled, for good reasons, on an 8 by 8 unstaggered grid over the semi-span.
2) Confusion about how to deal numerically with the singularity that occurs in the governing integral equations. One option is to try to avoid the problem by staggering the grid of the field points with respect to the grid of the control points. Instead of this, we have resolved the indeterminacy by rigorous analysis and as one consequence, are able to employ a simple unstaggered grid.
3) Confusion about how to evaluate the partial derivatives of the circulation function, whether analytically or by finite difference formulas. It was found that analytical differentiation is incorrect and that finite differences must be used.
4) Confusion about the validity of representing the circulation by a Fourier series. While this procedure is widely advocated in the technical literature, we found that a much simpler and clearer formulation can be obtained otherwise.
5) Confusion about the boundary conditions. Special conditions apply to the leading and trailing edges, the wing tip and at midspan. These quite complex boundary conditions have now been fully and rigorously analyzed and incorporated consistently into our formulation of the problem.

It is evident from the foregoing discussion that this research has amounted to a major education in basic aerodynamics and in numerical methods. In these respects it has been a deeply rewarding experience. However, in view of the foregoing obstacles and problems, the time schedule has of course been greatly delayed from that which was initially anticipated. Thus there are no final numerical results at this particular stage of the investigation and there is no more time available to the present research team. Hence, this final aspect will have to be completed by some subsequent investigators. Nevertheless, what the present effort has produced to bequeath to any subsequent worker is a sophisticated and refined numerical method. This method has now evolved to the point that it can be confidently expected to produce a reasonable and accurate final result.

II. INTRODUCTION

This thesis is the development of a computational method for determining the aerodynamic parmeters of wings in incompressible flow, from the theory developed by Jones [Ref. 1]. In conjunction with Jones' theory this analysis has solved the problems requiring further attention pointed out by Parks [Ref. 2]. Primarily a new approach, defining the circulation directly instead of using a Fourier series representation, is employed. This along with a direct method of dealing with the singularities that are encountered in the governing integral equations when a control point and field point coincide, allows the computational grids for those points to be identical. This analysis is intended to complement that of Jones by taking his final theoretical equations and, where applicable, presenting them in matrix format. The equations are then organized into a computational sequence which can be further translated into programming language. It is expected that a program to produce those wing parameters described by Jones will follow from this development.

III. MATRIX NOTATION

Though equations using subscripted variables are a primary method of portraying mathematical relations, it may be easier to visualize what is happening in those equations when they are written in matrix format. In this chapter those equations of reference 1 which lend themselve to this type of notation are rewritten in matrix format. The subscripts (indices) i,j,k are described for the wing in the $\phi\theta$ plane as shown in figure 3.1.

It will be convenient at many points throughout this presentation to transform two dimensional matrices into vectors and conversly vectors into two dimensional matrices. For doing this the following sybolic relations are defined;

$$\underset{64\times1}{\left\{\quad\right\}} \Rightarrow \underset{8\times8}{\left[\quad\right]}, \quad \underset{8\times8}{\left[\quad\right]} \Rightarrow \underset{64\times1}{\left\{\quad\right\}} \tag{3.1}$$

Figure 3.1 as it is drawn represents a two dimensional matrix with idices (J,I) and the index K shows the relation of a column vector that has been transformed into a matrix. The inverse operation can also be deduced from this figure.

A. CIRCULATION AND PRESSURE RELATIONS

The following equations are the general circulation derivatives and pressure relations. Those equations taken from reference 1 are noted by their equation number preceded by the letter J. Therefore (J6.8) becomes;

$$\left(\frac{\partial\Gamma}{\partial\phi}\right)_{j,i} = \underset{8\times8}{\left[\Gamma_\phi\right]} \Rightarrow \underset{64\times1}{\left\{\Gamma_\phi\right\}} \tag{3.2}$$

Similarly for (J6.9)

	$i=1$	$i=2$	$i=3$	$i=4$	$i=5$	$i=6$	$i=7$	$i=8$
$j=1$	$k=1$	$k=9$	$k=17$	-	-	-	-	$k=57$
$j=2$	$k=2$	$k=10$	-	-	-	-	-	$k=58$
$j=3$	$k=3$	$k=11$	-	-	-	-	-	$k=59$
$j=4$	$k=4$	$k=12$	-	-	-	-	-	$k=60$
$j=5$	$k=5$	$k=13$	-	-	-	-	-	$k=61$
$j=6$	$k=6$	$k=14$	-	-	-	-	-	$k=62$
$j=7$	$k=7$	$k=15$	-	-	-	-	-	$k=63$
$j=8$	$k=8$	$k=16$	$k=24$	-	-	-	-	$k=64$

Figure 3.1 Index notation.

$$\left(\frac{\partial\Gamma}{\partial\theta}\right)_{j,i} = \underset{8\times8}{\left[\Gamma_{\theta}\right]} \Rightarrow \underset{64\times1}{\left\{\Gamma_{\theta}\right\}} \tag{3.3}$$

Along the leading and trailing edges equations (J6.10) through (J6.13) respectively are

$$\left(\frac{\partial\Gamma}{\partial\phi}\right)_{L\,j} = \underset{8\times1}{\left\{\Gamma_{\phi L}\right\}} \tag{3.4}$$

$$\left(\frac{\partial\Gamma}{\partial\phi}\right)_{T\,j} = \underset{8\times1}{\left\{\Gamma_{\phi T}\right\}} \tag{3.5}$$

$$\left(\frac{\partial^2\Gamma}{\partial\phi^2}\right)_{L\,j} = \underset{8\times1}{\left\{\Gamma_{\phi\phi L}\right\}} \tag{3.6}$$

$$\left(\frac{\partial^2\Gamma}{\partial\phi^2}\right)_{T\,j} = \underset{8\times1}{\left\{\Gamma_{\phi\phi T}\right\}} \tag{3.7}$$

The pressure distribution (J6.14) is

$$\Delta C_p)_{j,i} = P(j,i) = \underset{8\times8}{\left[P\right]} \tag{3.8}$$

At the leading and trailing edges (J6.15) and (J6.16) are

$$\Delta C_p)_{Lj} = P_L(j) = \underset{8\times1}{\{P_L\}} \qquad (3.9)$$

$$\Delta C_p)_{Tj} = P_T(j) = \underset{8\times1}{\{P_T\}} \qquad (3.10)$$

Generally the pressure equation (J6.22) can be written;

$$\underset{8\times8}{\Big[\,P\,\Big]} = \frac{8}{A\!\!R}\underset{8\times8}{\Big[\diagdown\frac{1}{C}\diagdown\Big]}\underset{8\times8}{\Big[\,\Gamma_\phi\,\Big]}\underset{8\times8}{\Big[\diagdown\frac{1}{\mathrm{SIN}\phi}\diagdown\Big]} \qquad (3.11)$$

or solving for the circulation derivative in the chordwise (ϕ) direction

$$\underset{8\times8}{\Big[\,\Gamma_\phi\,\Big]} = \frac{A\!\!R}{8}\underset{8\times8}{\Big[\diagdown C\diagdown\Big]}\underset{8\times8}{\Big[\,P\,\Big]}\underset{8\times8}{\Big[\diagdown\mathrm{SIN}\phi\diagdown\Big]} \qquad (3.12)$$

B. CALCULATION OF DERIVATIVES

1. Chordwise Derivatives

Equations (J7.9), (J7.16) and (J7.14) are written in the index notation as follows

$$\Gamma_\phi(j,1) = \frac{1}{\Delta\phi}\left[\Gamma(j,1) + \frac{1}{3}\Gamma(j,2)\right] \qquad (3.13)$$

$$\Gamma_\phi(j,i) = \frac{1}{\Delta\phi}\left[\frac{-1}{2}\Gamma(j,i-1) + \frac{1}{2}\Gamma(j,i+1)\right] \qquad i=2,3,\ldots,7 \qquad (3.14)$$

$$\Gamma_\phi(j,8) = \frac{1}{\Delta\phi}\left[\frac{-3}{13}\Gamma(j,7) + \frac{3}{13}\Gamma(j,8)\right] \qquad (3.15)$$

This helps to visualize the equation in the matrix notation below

$$\underset{8\times8}{\Big[\,\Gamma_\phi\,\Big]^T} = \underset{8\times8}{\Big[\,D_a\,\Big]}\underset{8\times8}{\Big[\,\Gamma\,\Big]^T} \qquad (3.16)$$

Note that this equation involves the transpose of both the Γ matrix and the Γ_ϕ matrix. A description of the matrix Da may be found in figure 3.2.

$$[D_a] = \frac{1}{\Delta\phi}\begin{bmatrix} 1 & +\frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{2} & 0 & +\frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & +\frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 & +\frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{2} & 0 & +\frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & +\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & +\frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{3}{13} & +\frac{3}{13} \end{bmatrix}$$

Figure 3.2 Matrix Da.

Equations 3.13 through 3.15 can also be written in a single index notation.

$$\Gamma_\phi(k) = \frac{1}{\Delta\phi}\left[\Gamma(k) + \frac{1}{3}\Gamma(k+8)\right] \qquad 1 \le k \le 8 \qquad (3.17)$$

$$\Gamma_\phi(k) = \frac{1}{\Delta\phi}\left[\frac{-1}{2}\Gamma(k-8) + \frac{1}{2}\Gamma(k+8)\right] \qquad 9 \le k \le 56 \qquad (3.18)$$

$$\Gamma_\phi(k) = \frac{1}{\Delta\phi}\left[\frac{-3}{13}\Gamma(k-8) + \frac{3}{13}\Gamma(k)\right] \qquad 57 \le k \le 64 \qquad (3.19)$$

In order to get adjacent points in the chordwise direction, the integer 8 must be added to or subtracted from the k index describing the point as can be seen in figure 3.1 The above equations now can be written in matrix notation

$$\underset{64\times 1}{\{\Gamma_\phi\}} = \underset{64\times 64}{[D_a^*]} \underset{64\times 1}{\{\Gamma\}} \qquad (3.20)$$

where the general configuration and description of the elements in the matrix Da^* are shown in figure 3.7 and table 1 respectively. Notice from the figure that all of the nonzero elements of the matrix Da^* lie along just three principal diagonals.

Since in the basic lift case the pressure at the leading edge is finite, an additional term appears in the equation for the circulation derivatives. That term involves the second derivative of Γ. Since Γ_ϕ is zero, equation (J6.28) for the pressure at the leading edge is,

$$\underset{8\times 1}{\left\{P_L\right\}} = \frac{8}{A\!\!R}\underset{8\times 8}{\left[\diagdown \frac{1}{c} \diagdown\right]}\underset{8\times 1}{\left\{\Gamma_{\phi\phi L}\right\}} \tag{3.21}$$

or solving for $\Gamma_{\phi\phi L}$

$$\underset{8\times 1}{\left\{\Gamma_{\phi\phi L}\right\}} = \frac{A\!\!R}{8}\underset{8\times 8}{\left[\diagdown c \diagdown\right]}\underset{8\times 1}{\left\{P_L\right\}} \tag{3.22}$$

By taking the elements of both $\{\Gamma_{\phi\phi L}\}$ and $\{P_L\}$ and placing them in the first column of the partitioned matrices with the same name thusly,

$$\underset{8\times 8}{\left[\Gamma_{\phi\phi L}\right]} = \left[\underset{8\times 1}{\left\{\Gamma_{\phi\phi L}\right\}} \,\vdots\, \underset{8\times 7}{\left[\,0\,\right]}\right] \tag{3.23}$$

$$\underset{8\times 8}{\left[P_L\right]} = \left[\underset{8\times 1}{\left\{P\right\}} \,\vdots\, \underset{8\times 7}{\left[\,0\,\right]}\right] \tag{3.24}$$

the vector equation can then be written in matrix form

$$\underset{8\times 8}{\left[\Gamma_{\phi\phi L}\right]} = \frac{A\!\!R}{8}\underset{8\times 8}{\left[\diagdown c \diagdown\right]}\underset{8\times 8}{\left[P_L\right]} \tag{3.25}$$

Now the Γ_ϕ equations, (J7.10), (J7.16) and (J7.14) are;

$$\frac{\Delta\phi}{4}\Gamma_{\phi\phi L}(j) + \Gamma_\phi(j,1) = \frac{1}{\Delta\phi}\left[6\Gamma(j,1)\right] \tag{3.26}$$

$$\Gamma_\phi(j,i) = \frac{1}{\Delta\phi}\left[\frac{-1}{2}\Gamma(j,i-1) + \frac{1}{2}\Gamma(j,i+1)\right] \qquad i=2,3,\ldots,7 \qquad (3.27)$$

$$\Gamma_\phi(j,8) = \frac{1}{\Delta\phi}\left[\frac{-3}{13}\Gamma(j,7) + \frac{3}{13}\Gamma(j,8)\right] \qquad (3.28)$$

for which the matrix notation is;

$$\frac{\Delta\phi}{4}\underset{8\times8}{\left[\Gamma_{\phi\phi L}\right]^T} + \underset{8\times8}{\left[\Gamma_\phi\right]^T} = \underset{8\times8}{\left[D_b\right]}\underset{8\times8}{\left[\Gamma\right]^T} \qquad (3.29)$$

or solving for circulation,

$$\underset{8\times8}{\left[\Gamma\right]^T} = \underset{8\times8}{\left[D_b\right]^{-1}}\left(\frac{\Delta\phi}{4}\left[\Gamma_{\phi\phi L}\right]^T + \left[\Gamma_\phi\right]^T\right) \qquad (3.30)$$

with the contents of Db shown in figure 3.3

$$\left[D_b\right] = \frac{1}{\Delta\phi}\begin{array}{|c|c|c|c|c|c|c|c|}\hline 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline \frac{-1}{2} & 0 & \frac{+1}{2} & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & \frac{-1}{2} & 0 & \frac{+1}{2} & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & \frac{-1}{2} & 0 & \frac{+1}{2} & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & \frac{-1}{2} & 0 & \frac{+1}{2} & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & \frac{-1}{2} & 0 & \frac{+1}{2} & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & \frac{-1}{2} & 0 & \frac{+1}{2} \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & \frac{-3}{13} & \frac{+3}{13} \\ \hline\end{array}$$

Figure 3.3 Matrix Db.

Notice that the element of Db are identical to those of Da except for the first two elements of the first row, which are 6 and 0 vice 1 and 1/3.

2. Spanwise Derivatives

For the derivatives in the spanwise (θ) direction a set of relations similar to those for Γ_ϕ can be shown. From (J7.17), (J7.18) and (J7.23) respectively

$$\Gamma_\theta(1,i) = \frac{1}{\Delta\theta}\left[\Gamma(1,i) + \frac{1}{3}\Gamma(2,i)\right] \tag{3.31}$$

$$\Gamma_\theta(j,i) = \frac{1}{\Delta\theta}\left[\frac{-1}{2}\Gamma(j-1,i) + \frac{1}{2}\Gamma(j+1,i)\right] \qquad j=2,3,\ldots,7 \tag{3.32}$$

$$\Gamma_\theta(8,i) = \frac{1}{\Delta\theta}\left[\frac{1}{2}\Gamma(6,i) - 2\Gamma(7,i) + \frac{3}{2}\Gamma(8,i)\right] \tag{3.33}$$

So that in matrix notation the equation is,

$$\left[\Gamma_\theta\right] = \left[E\right]\left[\Gamma\right] \tag{3.34}$$

and the E matrix is defined in figure 3.4.

$$\left[E\right] = \frac{1}{\Delta\theta}\begin{array}{|c|c|c|c|c|c|c|c|}
\hline
1 & \frac{+1}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
\frac{-1}{2} & 0 & \frac{+1}{2} & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & \frac{-1}{2} & 0 & \frac{+1}{2} & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & \frac{-1}{2} & 0 & \frac{+1}{2} & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & \frac{-1}{2} & 0 & \frac{+1}{2} & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & \frac{-1}{2} & 0 & \frac{+1}{2} & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & \frac{-1}{2} & 0 & \frac{+1}{2} \\
\hline
0 & 0 & 0 & 0 & 0 & \frac{+1}{2} & -2 & \frac{+3}{2} \\
\hline
\end{array}$$

Figure 3.4 Matrix E.

In a manner similar to the Γ_ϕ equations, equations 3.31 through 3.33 can be written in single index notation

$$\Gamma_\theta(k)=\frac{1}{\Delta\theta}\left[\Gamma(k)+\frac{1}{3}\Gamma(k+1)\right] \qquad k=1,9,17,\ldots,57 \qquad (3.35)$$

$$\Gamma_\theta(k)=\frac{1}{\Delta\theta}\left[\frac{-1}{2}\Gamma(k-1)+\frac{1}{2}\Gamma(k+1)\right] \qquad k=2\text{--}7,10\text{--}15,\ldots,58\text{--}63 \qquad (3.36)$$

$$\Gamma_\theta(k)=\frac{1}{\Delta\theta}\left[\frac{1}{2}\Gamma(k-2)-2\Gamma(k-1)+\frac{3}{2}\Gamma(k)\right] \qquad k=8,16,24,\ldots,64 \qquad (3.37)$$

These equations when viewed in the vector form are much neater.

$$\underset{64\times1}{\left\{\Gamma_\theta\right\}}=\underset{64\times64}{\left[E^*\right]}\underset{64\times1}{\left\{\Gamma\right\}} \qquad (3.38)$$

The general configuration of the E* matrix along with the description of its elements is shown in figure 3.8 and table 2. Notice from the figure that all of the nonzero elements of matrix E* lie along just four principal diagonals. Storage can be minimized by storing [Da*] and [Db*] as 64 by 3 matrices and [E*] as a 64 by 4 matrix, capitalizing on the large number of zeros in each.

It is useful to consider the hypothetical case in which the circulation function is assigned unit value at an arbitrary point Q (the 'circulation point') with matrix indices JQ and IQ or equivalent vector index KQ, and is assigned zero values at all other points of the calculation grid.

The above hypothetical distribution of Γ implies that of the many field points (denoted by index K) there will be certain ones adjacent to the circulation point Q at which the derivatives Γ_ϕ and Γ_θ take on nonzero values. In the simplest and most typical case where Q does not lie adjacent to any of the boundaries, there will be two field points, at locations K=(KQ-8) and K=(KQ+8), at which Γ_ϕ takes on the values $+1/2\Delta\phi$ and $-1/2\Delta\phi$. Likewise, there will be two other points, at locations K=(KQ-1) and K=(KQ+1), at which Γ_θ takes on the values $+1/2\Delta\theta$ and $-1/2\Delta\theta$. This situation is illustrated by the schematic diagram in figure 3.5.

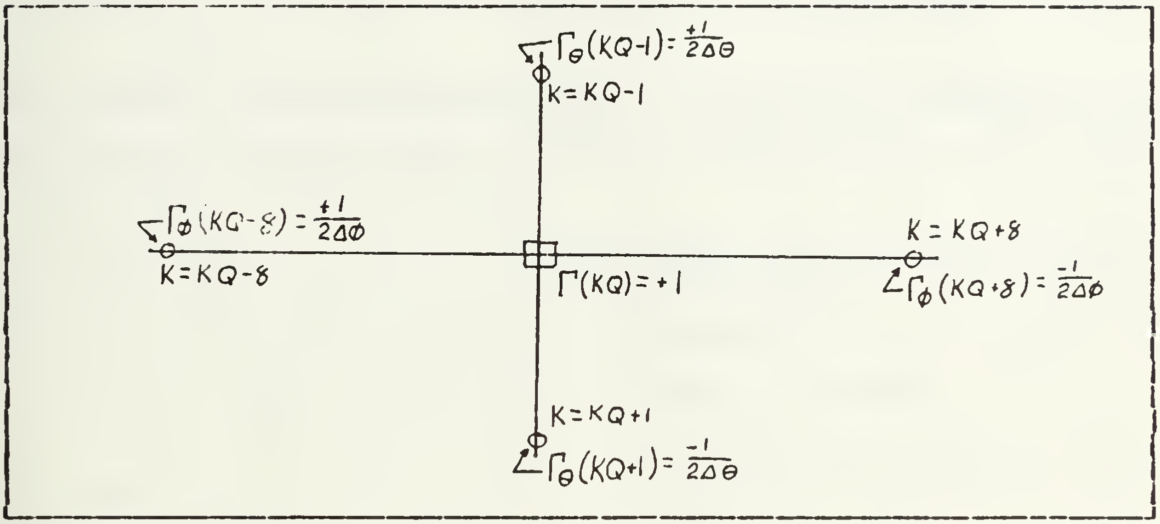

Figure 3.5 Circulation point.

More complicated situations arise if the point Q lies immediately adjacent to a boundary. In this case the number of field points having nonzero values of Γ_ϕ and Γ_θ may range from three to five. The details can be seen from careful study of the matrices Da*, Db* and E*.

Along the trailing edge a set of relations for the circulation function and its spanwise derivative is required. From a similar analysis to that used for the spanwise derivative over the wing, the following set of equation for the trailing edge is obtained.

$$\Gamma_{\theta T}(1) = \frac{1}{\Delta\theta}\left[\Gamma_T(1) + \frac{1}{3}\Gamma_T(2)\right] \qquad (3.39)$$

$$\Gamma_{\theta T}(j) = \frac{1}{\Delta\theta}\left[\frac{-1}{2}\Gamma_T(j-1) + \frac{1}{2}\Gamma_T(j+1)\right] \qquad j=2,3,\ldots,7 \qquad (3.40)$$

$$\Gamma_{\theta T}(8)=\frac{1}{\Delta\theta}\left[\frac{+1}{2}\Gamma_T(6)-2\Gamma_T(7)+\frac{3}{2}\Gamma_T(8)\right] \qquad (3.41)$$

This set of equations can be written as a single matrix equation

$$\underset{8\times 1}{\{\Gamma_{\theta T}\}}=\underset{8\times 8}{[E]}\,\underset{8\times 1}{\{\Gamma_T\}} \qquad (3.42)$$

This E matrix is the same as the one described earlier in figure 3.4. From equation (J7.15) at the trailing edge

$$\Gamma_T(j)=-\frac{1}{26}\Gamma(j,7)+\frac{27}{26}\Gamma(j,8) \qquad (3.43)$$

Rewritten as a vector equation

$$\underset{8\times 1}{\{\Gamma_T\}}=\frac{-1}{26}\underset{8\times 1}{\begin{Bmatrix}\Gamma(49)\\ \vdots\\ \Gamma(56)\end{Bmatrix}}+\frac{27}{26}\underset{8\times 1}{\begin{Bmatrix}\Gamma(57)\\ \vdots\\ \Gamma(64)\end{Bmatrix}} \qquad (3.44)$$

Notice that of the 64 elements in the Γ vector only the last 16 are used.

C. INTEGRAL EQUATION FOR WING SLOPE

1. First Wing Slope Function

The integral equation (J7.24), is restated here as equation 3.45

$$W_P'=\frac{1}{2\pi AR}\int_0^{\pi/2}\int_0^{\pi}\left\{\left[\frac{F_1}{r_1^3}+\frac{F_2}{r_2^3}\right]SIN\theta\,\Gamma_\phi-\left[\frac{(\eta-\eta_P)}{r_1^3}+\frac{(\eta+\eta_P)}{r_2^3}\right]\frac{C}{2}SIN\phi\,\Gamma_\theta\right\}d\phi\,d\theta \qquad (3.45)$$

By defining the following two relations

$$T(k_P,k)=\frac{\Delta\phi\,\Delta\theta}{2\pi AR}\left[\frac{F_1}{r_1^3}+\frac{F_2}{r_2^3}\right]SIN\theta \qquad (3.46)$$

and

$$S(k_p,k) = \frac{\Delta\phi\Delta\theta}{2\pi AR}\left[\frac{(\eta-\eta_p)}{r_1^3} + \frac{(\eta+\eta_p)}{r_2^3}\right]\frac{C}{2}\sin\phi \tag{3.47}$$

the integral equation can be written in index notation as

$$W_p'(k_p) = \left(\sum_{k_Q=1}^{64}\sum_{k=1}^{64}\Big(T(k_p,k)\,D^*(k,k_Q) - S(k_p,k)\,E^*(k,k_Q)\Big)\right)\Gamma(k_p) \tag{3.48}$$

The matrix D^* represents either Da^* or Db^* whichever is appropriate. The elements of Da^* and Db^* are shown in table 1 and those of E^* in table 2.

To make the distinction between additional and basic lift the following two matrices are defined

$$A'(k_p,k_Q) = \sum_{k=1}^{64}\Big(T(k_p,k)\,D_a^*(k,k_Q) - S(k_p,k)\,E^*(k,k_Q)\Big) \tag{3.49}$$

$$B'(k_p,k_Q) = \sum_{k=1}^{64}\Big(T(k_p,k)\,D_b^*(k,k_Q) - S(k_p,k)\,E^*(k,k_Q)\Big) \tag{3.50}$$

The above caculation for [A'] and [B'] is simpler than the 64 by 64 multiplications indicated. If one uses tables 1 and 2 the products are simplified to,

$$\begin{aligned}\sum_{k=1}^{64} T(k_p,k)\,D_a^*(k,k_Q) = {} & T(k_p,k_Q-8)\,D_a^*(k_Q-8,k_Q)\\ & + T(k_p,k_Q)\,D_a^*(k_Q,k_Q)\\ & + T(k_p,k_Q+8)\,D_a^*(k_Q+8,k_Q)\end{aligned} \tag{3.51}$$

$$\begin{aligned}\sum_{k=1}^{64} T(k_p,k)\,D_b^*(k,k_Q) = {} & T(k_p,k_Q-8)\,D_b^*(k_Q-8,k_Q)\\ & + T(k_p,k_Q)\,D_b^*(k_Q,k_Q)\\ & + T(k_p,k_Q+8)\,D_b^*(k_Q+8,k_Q)\end{aligned} \tag{3.52}$$

$$\begin{aligned}\sum_{k=1}^{64} S(k_p,k)\,E^*(k,k_Q) = {} & S(k_p,k_Q-1)\,E^*(k_Q-1,k_Q)\\ & + S(k_p,k_Q)\,E^*(k_Q,k_Q)\end{aligned}$$

$$\begin{aligned} &+ S(kP,kQ+1)E^*(kQ+1,kQ) \\ &+ S(kP,kQ+2)E^*(kQ+2,kQ) \end{aligned} \qquad (3.53)$$

The calculation of [A'] and [B'] then is

$$\begin{aligned} A'(kP,kQ) = {} & T(kP,kQ-8)D_a^*(kQ-8,kQ) \\ &+ T(kP,kQ)D_a^*(kQ,kQ) \\ &+ T(kP,kQ+8)D_a^*(kQ+8,kQ) \\ &- S(kP,kQ-1)E^*(kQ-1,kQ) \\ &- S(kP,kQ)E^*(kQ,kQ) \\ &- S(kP,kQ+1)E^*(kQ+1,kQ) \\ &- S(kP,kQ+2)E^*(kQ+2,kQ) \end{aligned} \qquad (3.54)$$

$$\begin{aligned} B'(kP,kQ) = {} & T(kP,kQ-8)D_b^*(kQ-8,kQ) \\ &+ T(kP,kQ)D_b^*(kQ,kQ) \\ &+ T(kP,kQ+8)D_b^*(kQ+8,kQ) \\ &- S(kP,kQ-1)E^*(kQ-1,kQ) \\ &- S(kP,kQ)E^*(kQ,kQ) \\ &- S(kP,kQ+1)E^*(kQ+1,kQ) \\ &- S(kP,kQ+2)E^*(kQ+2,kQ) \end{aligned} \qquad (3.55)$$

The various factors that occur in the above equations take on definite numerical values if and only if the indices KP and KQ are first assigned definite numerical values. When this is done the above equations define the value of a single element of either [A'] or [B']. The matrices [A'] and [B'] can be stored as two 64 by 64 arrays. So the matrix equations for the first wing slope function are, for the additional and basic lift cases, respectively,

$$\underset{64\times 1}{\{W_P'\}} = \underset{64\times 64}{[A']}\ \underset{64\times 1}{\{\Gamma\}} \qquad (3.56)$$

$$\underset{64\times 1}{\{W_p'\}} = \underset{64\times 64}{[B']}\,\underset{64\times 1}{\{\Gamma\}} \qquad (3.57)$$

2. Second Wing Slope Function

The integral equation for the second wing slope function (J7.25) is

$$W_p'' = \frac{-1}{2\pi AR}\int_0^{\pi/2}\left[\frac{G_1}{(\eta-\eta_p)}+\frac{G_2}{(\eta+\eta_p)}\right]\Gamma_{\theta T}\,d\theta \qquad (3.58)$$

This equation requires only the derivative of the circulation function in the θ direction along the trailing edge. Therefore a relation involving the same E matrix as before can be utilized. By defining a function

$$H(k_p, j) = \frac{-\Delta\theta}{2\pi AR}\left(\frac{G_1}{(\eta-\eta_p)}+\frac{G_2}{(\eta+\eta_p)}\right) \qquad (3.59)$$

equation 3.58 can be written in index notation as

$$W_p''(k_p) = \sum_{j=1}^{8} H(k_p, j)\,\Gamma_{\theta T}(j) \qquad (3.60)$$

In matrix notation it becomes

$$\underset{64\times 1}{\{W_p''\}} = \underset{64\times 8}{[H]}\,\underset{8\times 1}{\{\Gamma_{\theta T}\}} \qquad (3.61)$$

Equation 3.42 can be substituted for $\Gamma_{\theta T}$ and the equation now becomes

$$\underset{64\times 1}{\{W_p''\}} = \underset{64\times 8}{[H]}\,\underset{8\times 8}{[E]}\,\underset{8\times 1}{\{\Gamma_T\}} \qquad (3.62)$$

Then substituting equation 3.44 for Γ_T gives

$$\underset{64\times 1}{\{W_p''\}} = \underset{64\times 8}{[H]}\,\underset{8\times 8}{[E]}\left(-\frac{1}{26}\underset{8\times 1}{\begin{Bmatrix}\Gamma(49)\\ \vdots \\ \Gamma(56)\end{Bmatrix}} + \frac{27}{26}\underset{8\times 1}{\begin{Bmatrix}\Gamma(57)\\ \vdots \\ \Gamma(64)\end{Bmatrix}}\right) \qquad (3.63)$$

For convenience an additional matrix is defined as follows,

$$\underset{64\times 8}{[Q]} = \underset{64\times 8}{[H]}\,\underset{8\times 8}{[E]} \qquad (3.64)$$

Now another matrix can be defined which shall be termed the V matrix. Matrix V is a partitioned matrix, which is divided into three parts as shown in figure 3.6.

$$\underset{64\times 64}{[V]} = \left[\ \underset{64\times 48}{\left[\quad 0 \quad\right]} \ \middle|\ \frac{-1}{26}\underset{64\times 8}{\left[\ Q\ \right]} \ \middle|\ \frac{+27}{26}\underset{64\times 8}{\left[\ Q\ \right]}\ \right]$$

Figure 3.6 Matrix V.

Therefore the final form of the integral equation is,

$$\underset{64\times 1}{\{W_p''\}} = \underset{64\times 64}{[V]}\,\underset{64\times 1}{\{\Gamma\}} \qquad (3.65)$$

3. Final Wing Slope Function

The two relations for Wp' and Wp" can now be superimposed to give the final equation for the wing slope function. From (J7.2)

$$\underset{64\times 1}{\{W_p\}} = \underset{64\times 1}{\{W_p'\}} + \underset{64\times 1}{\{W_p''\}} \qquad (3.66)$$

After substituting the relations for Wp' and Wp" and simpifying, the equations become

$$\underset{64\times 1}{\{W_p\}} = \left(\underset{64\times 64}{[A']} + \underset{64\times 64}{[V]}\right)\underset{64\times 1}{\{\Gamma\}} \qquad (3.67)$$

$$\underset{64\times1}{\{W_P\}} = \left(\underset{64\times64}{[B']} + \underset{64\times64}{[V]}\right)\underset{64\times1}{\{\Gamma\}} \tag{3.68}$$

By defining two new matrices [A] and [B] as follows,

$$\underset{64\times64}{[A]} = \underset{64\times64}{[A']} + \underset{64\times64}{[V]} \tag{3.69}$$

$$\underset{64\times64}{[B]} = \underset{64\times64}{[B']} + \underset{64\times64}{[V]} \tag{3.70}$$

the final wing slope functions can be written as

$$\underset{64\times1}{\{W_P\}} = \underset{64\times64}{[A]}\,\underset{64\times1}{\{\Gamma\}} \tag{3.71}$$

for the additional lift, or as

$$\underset{64\times1}{\{W_P\}} = \underset{64\times64}{[B]}\,\underset{64\times1}{\{\Gamma\}} \tag{3.72}$$

for the basic lift. The additional lift equation can be inverted as follows to be used in later calculations

$$\underset{64\times1}{\{\Gamma\}} = \underset{64\times64}{[A]^{-1}}\,\underset{64\times1}{\{W_P\}} \tag{3.73}$$

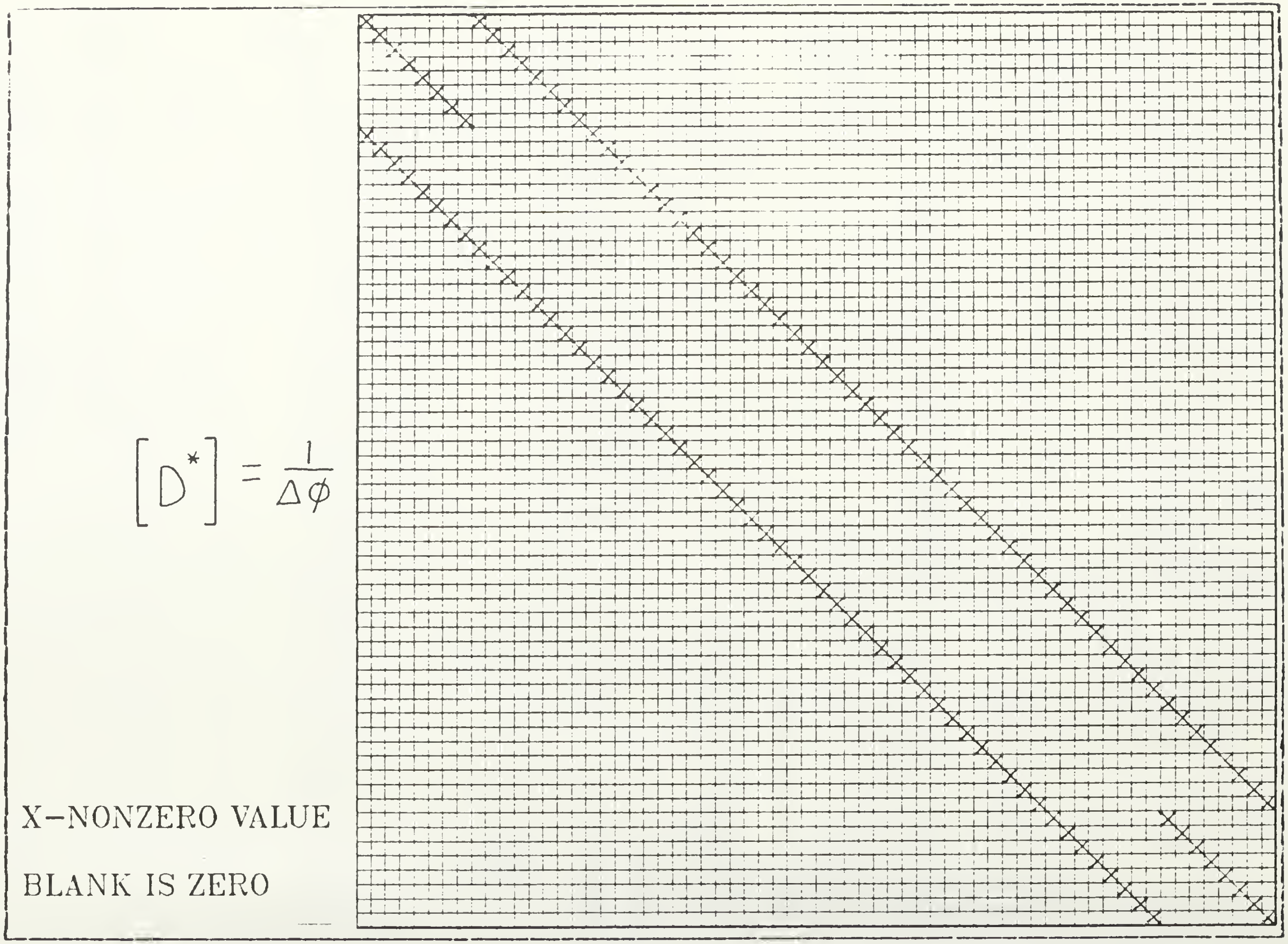

Figure 3.7 Diagonal structure of Da*and Db*.

TABLE 1

Elements of Da* and Db* Along the Three Principal Diagonals

Da*

KQ	1 -- 8	9 -- 16	17 -- 48	49 -- 56	57 -- 64
K=KQ-8	0	+1/3	+1/2	+1/2	+1/2
K=KQ	1	0	0	0	+3/13
K=KQ+8	-1/2	-1/2	-1/2	-3/13	0

Db*

KQ	1 -- 8	9 -- 16	17 -- 48	49 -- 56	57 -- 64
K=KQ-8	0	0	+1/2	+1/2	+1/2
K=KQ	6	0	0	0	+3/13
K=KQ+8	-1/2	-1/2	-1/2	-3/13	0

Numerical coefficients shown must be divided by $\Delta\phi$

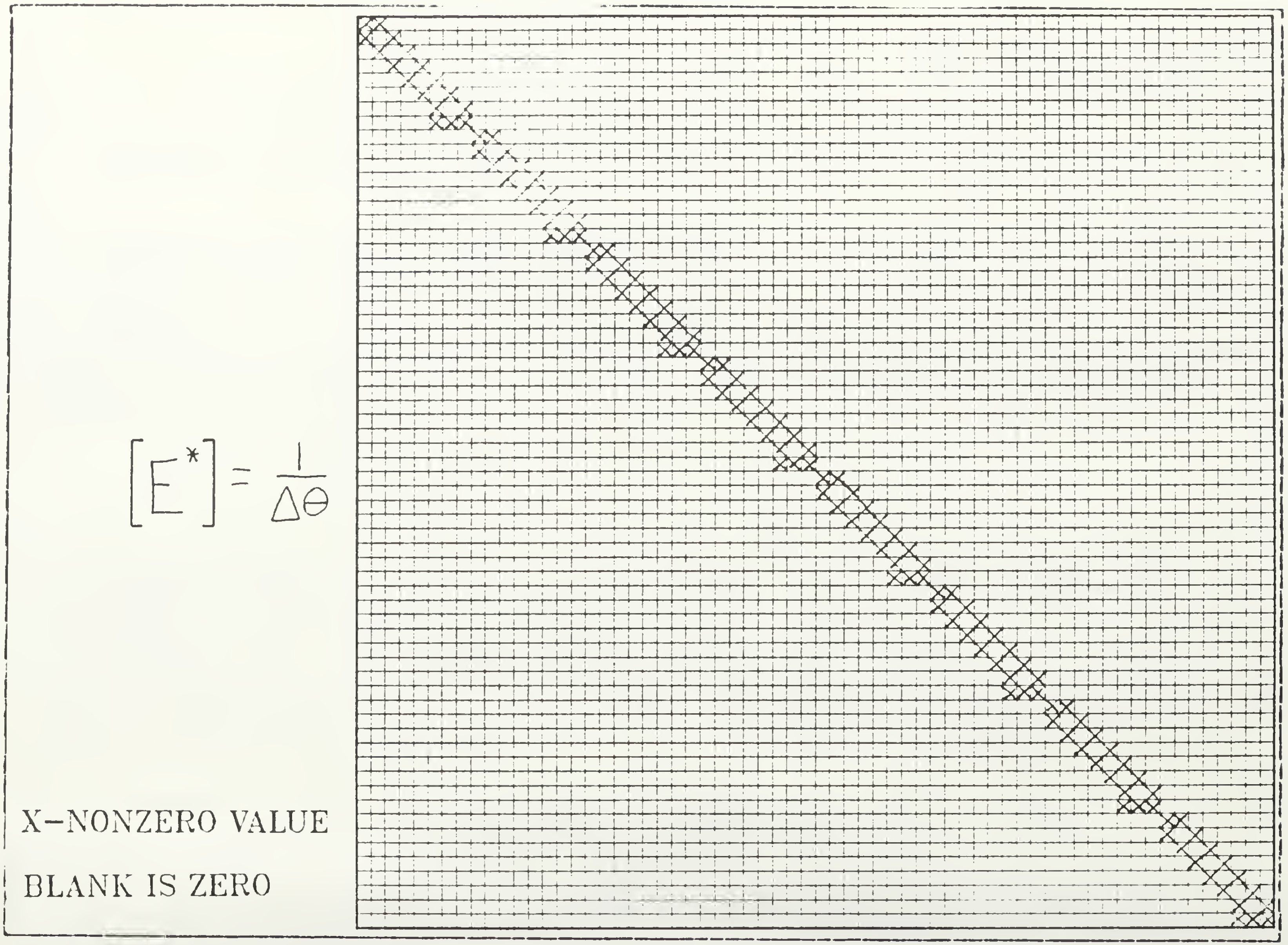

Figure 3.8 Diagonal Structure of E^*.

TABLE 2

Elements of E^*
Along the Four Principal Diagonals

KQ	1	2	3,4,5	6	7	8	9	10	11,12,13	14	15	16
K=KQ-1	0	+1/3	+1/2	+1/2	+1/2	+1/2	0	+1/3	+1/2	+1/2	+1/2	+1/2
K=KQ	1	0	0	0	0	+3/2	1	0	0	0	0	+3/2
K=KQ+1	-1/2	-1/2	-1/2	-1/2	-2	0	-1/2	-1/2	-1/2	-1/2	-2	0
K=KQ+2	0	0	0	+1/2	0	0	0	0	0	+1/2	0	0

KQ	17	18	19,20,21	22	23	24	25	26	27,28,29	30	31	32
K=KQ-1	0	+1/3	+1/2	+1/2	+1/2	+1/2	0	+1/3	+1/2	+1/2	+1/2	+1/2
K=KQ	1	0	0	0	0	+3/2	1	0	0	0	0	+3/2
K=KQ+1	-1/2	-1/2	-1/2	-1/2	-2	0	-1/2	-1/2	-1/2	-1/2	-2	0
K=KQ+2	0	0	0	+1/2	0	0	0	0	0	+1/2	0	0

Numerical coefficients shown must be divided by $\Delta\theta$

Table 2

Elements of E*

Along the Four Principal Diagonals (cont'd)

KQ	33	34	35,36,37	38	39	40	41	42	43,44,45	46	47	48
K=KQ-1	0	+1/3	+1/2	+1/2	+1/2	+1/2	0	+1/3	+1/2	+1/2	+1/2	+1/2
K=KQ	1	0	0	0	0	+3/2	1	0	0	0	0	+3/2
K=KQ+1	-1/2	-1/2	-1/2	-1/2	-2	0	-1/2	-1/2	-1/2	-1/2	-2	0
K=KQ+2	0	0	0	+1/2	0	0	0	0	0	+1/2	0	0

KQ	49	50	51,52,53	54	55	56	57	58	59,60,61	62	63	64
K=KQ-1	0	+1/3	+1/2	+1/2	+1/2	+1/2	0	+1/3	+1/2	+1/2	+1/2	+1/2
K=KQ	1	0	0	0	0	+3/2	1	0	0	0	0	+3/2
K=KQ+1	-1/2	-1/2	-1/2	-1/2	-2	0	-1/2	-1/2	-1/2	-1/2	-2	0
K=KQ+2	0	0	0	+1/2	0	0	0	0	0	+1/2	0	0

Numerical coefficients shown must be divided by $\Delta\theta$

IV. SUMMARY OF KEY RELATIONSHIPS

A. PRELIMINARY CALCULATIONS

The initial set of calculations to be performed by the computer are for the values of those constants which need only be calculated once. The following equations use only the input parameters $\mathcal{AR}$, λ, τ and the indices i and j.

$$\Delta\phi = \pi/8 \tag{4.1}$$

$$\Delta\theta = \pi/16 \tag{4.2}$$

$$C' = \frac{\Delta\phi\Delta\theta}{2\pi\mathcal{AR}} \tag{4.3}$$

$$C'' = \frac{\Delta\theta}{2\pi\mathcal{AR}} \tag{4.4}$$

The following equations involve vectors whose indices i or j go from 1 to 8.

$$\phi(i) = \Delta\phi(i-\tfrac{1}{2}) \tag{4.5}$$

$$\theta(j) = \Delta\theta(j-\tfrac{1}{2}) \tag{4.6}$$

$$\xi(i) = \tfrac{1}{2}(1-\cos(\phi(i))) \tag{4.7}$$

$$\eta(j) = \cos(\theta(j)) \tag{4.8}$$

$$C(j) = \frac{2}{\mathcal{AR}}((1+\tau)-2\tau\eta(j)) \tag{4.9}$$

$$\mu(i) = \lambda - \frac{4\tau}{\mathcal{AR}}\xi(i) \tag{4.10}$$

$$\phi_L(j) = 0 \tag{4.11}$$

$$\phi_T(j) = \pi \tag{4.12}$$

$$\theta_T(i) = 0 \tag{4.13}$$

$$\theta_m(i) = \pi/2 \tag{4.14}$$

Indices i and j can be obtained from the index k through the two following integer equations

$$i = (k/8) + 1 \quad \text{(integer division)} \tag{4.15}$$

$$j = k - 8(i-1) \tag{4.16}$$

Indices ip and jp can be obtained from kp similarly.

B. CALCULATION OF THE A AND B MATRICES

1. The A' and B' Matrices

Since the matrices [A] and [B] are the sum of the [V] and [A'] or [B'] matrices, each of the matrices will be computed separately and then brought together to form [A] and [B]. Recall the equations

$$[A'] = [T][D_a^*] - [S][E^*] \tag{4.17}$$

and

$$[B'] = [T][D_b^*] - [S][E^*] \tag{4.18}$$

Where [T], [S], [D*] and [E*] have been previously defined.

It is convenient to make the following variable definitions

$$\Delta \eta_1 = (\eta - \eta_P) \tag{4.19}$$

$$\Delta \eta_2 = (\eta + \eta_P) \tag{4.20}$$

$$\Delta X = \lambda(\Delta \eta_1) + C\xi - C_P \xi_P \tag{4.21}$$

$$r_1 = \sqrt{\Delta X^2 + \Delta \eta_1^2} \tag{4.22}$$

$$r_2 = \sqrt{\Delta X^2 + \Delta \eta_2^2} \tag{4.23}$$

$$F_1 = \Delta X - \Delta \eta_1 \mu \tag{4.24}$$

$$F_2 = \Delta X - \Delta \eta_2 \mu \tag{4.25}$$

$$T(kp, k) = C'\left(\frac{F_1}{r_1^3} + \frac{F_2}{r_2^3}\right) SIN\,\Theta(j) \tag{4.26}$$

$$S(kp, k) = C'\left(\frac{\Delta \eta_1}{r_1^3} + \frac{\Delta \eta_2}{r_2^3}\right) \frac{C(j)}{2} SIN\,\phi(i) \tag{4.27}$$

$$T(kp, kp) = C'\left(\frac{F_2}{r_2^3}\right) SIN\,\Theta(jp) \tag{4.28}$$

$$S(kp, kp) = C'\left(\frac{\Delta \eta_2}{r_2^3}\right) \frac{C(jp)}{2} SIN\,\phi(ip) \tag{4.29}$$

A check should be made to see whether the control point and field point coincide. If they do, then equations 4.28 and 4.29 shall be used instead of 4.26 and 4.27 The matrices [A'] and [B'] from equations 3.54 and 3.55 are

$$\begin{aligned} A'(kP, kQ) = {} & T(kP, kQ-8)\, D_a^*(kQ-8, kQ) \\ & + T(kP, kQ)\, D_a^*(kQ, kQ) \\ & + T(kP, kQ+8)\, D_a^*(kQ+8, kQ) \\ & - S(kP, kQ-1)\, E^*(kQ-1, kQ) \\ & - S(kP, kQ)\, E^*(kQ, kQ) \\ & - S(kP, kQ+1)\, E^*(kQ+1, kQ) \\ & - S(kP, kQ+2)\, E^*(kQ+2, kQ) \end{aligned} \tag{4.30}$$

$$
\begin{aligned}
B'(kp,kQ) = {} & T(kp,kQ-8)\,D_b^*(kQ-8,kQ) \\
& + T(kp,kQ)\,D_b^*(kQ,kQ) \\
& + T(kp,kQ+8)\,D_b^*(kQ+8,kQ) \\
& - S(kp,kQ-1)\,E^*(kQ-1,kQ) \\
& - S(kp,kQ)\,E^*(kQ,kQ) \\
& - S(kp,kQ+1)\,E^*(kQ+1,kQ) \\
& - S(kp,kQ+2)\,E^*(kQ+2,kQ)
\end{aligned}
\tag{4.31}
$$

The various factors that occur in the above equations take on definite numerical values if and only if the indices KP and KQ are first assigned definite numerical values. When this is done the above equations define the value of a single element of either [A'] or [B']. The matrices [A'] and [B'] can be stored as two 64 by 64 arrays.

2. The V Matrix

Recall figure 3.6 with the associated equation 3.64 repeated here to provide clarity.

$$
\underset{64\times 64}{\Big[\,V\,\Big]} = \Big[\underset{64\times 48}{\Big[\,0\,\Big]} \,\Big\vert\, \frac{-1}{26}\underset{64\times 8}{\Big[\,Q\,\Big]} \,\Big\vert\, \frac{+27}{26}\underset{64\times 8}{\Big[\,Q\,\Big]}\Big]
\tag{4.32}
$$

$$
\underset{64\times 8}{\Big[\,Q\,\Big]} = \underset{64\times 8}{\Big[\,H\,\Big]}\underset{8\times 8}{\Big[\,E\,\Big]}
\tag{4.33}
$$

In addition to selected definitions above these additional definitions are made

$$
\Delta X_T = \lambda(\Delta\eta_1) + C - C_p\mathcal{E}_{\eta p}
\tag{4.34}
$$

$$
r_{T1} = \sqrt{\Delta X_T^{\,2} + \Delta\eta_1^{\,2}}
\tag{4.35}
$$

$$
r_{T2} = \sqrt{\Delta X_T^{\,2} + \Delta\eta_2^{\,2}}
\tag{4.36}
$$

$$
G_1 = 1 - \frac{\Delta X_T}{r_{T1}}
\tag{4.37}
$$

$$G_2 = 1 - \frac{\Delta X_T}{r_{T2}} \qquad (4.38)$$

$$H(k_P, j) = c''\left(\frac{G_1}{\Delta\eta_1} + \frac{G_2}{\Delta\eta_2}\right) \qquad (4.39)$$

$$H(k_P, j) = c''\left(\frac{G_2}{\Delta\eta_2}\right) \qquad (4.40)$$

A check should be made to see whether the spanwise coordinates coincide. If they do then equation 4.40 shall be used instead of 4.39 Then [Q] can be computed

$$Q(k_P, i) = \sum_{j=1}^{8} H(k_P, j)\, E(j, i) \qquad (4.41)$$

Now the [V] matrix can be constructed as was shown in figure 3.6. Since the first 48 columns of V are zeros, only the nonzero elements of V need be stored. These amount to a 64 by 16 matrix.

3. The A and B Matrices

Finally these matrices are superimposed to give the A and B matrices.

$$A(k_P, k_Q) = A'(k_P, k_Q) + V(k_P, k_Q) \qquad (4.42)$$

$$B(k_P, k_Q) = B'(k_P, k_Q) + V(k_P, k_Q) \qquad (4.43)$$

This gives the complete A and B matrices, which will be utilized in the following calculations.

C. ADDITIONAL LIFT

For the specific additional lift, as explained in more detail by Jones [Ref. 1], setting the normalized slope to -1, gives

$$\underset{64\times 1}{\{\Gamma\}} = \underset{64\times 64}{[A]}^{-1} \underset{64\times 1}{\begin{Bmatrix} -1 \\ \vdots \\ -1 \end{Bmatrix}} \tag{4.44}$$

This circulation can then be transformed by the operation described earlier to get

$$\underset{64\times 1}{\{\Gamma\}} \Rightarrow \underset{8\times 8}{[\Gamma]} \tag{4.45}$$

After being transposed it can be used in equation 3.16 to get the chordwise derivative

$$\underset{8\times 8}{[\Gamma_\phi]}^T = \underset{8\times 8}{[D_a]} \underset{8\times 8}{[\Gamma]}^T \tag{4.46}$$

The corresponding derivative in the θ direction is not generally needed and is not shown here, but has been stated earlier as equation 3.34. Now $[\Gamma_\phi]^T$ is transposed and substituted into equation 3.11 to get the pressure distribution.

$$\underset{8\times 8}{[P]} = \frac{8}{A\!\!R} \underset{8\times 8}{\left[\diagdown \frac{1}{c} \diagdown\right]} \underset{8\times 8}{[\Gamma_\phi]} \underset{8\times 8}{\left[\diagdown \frac{1}{\sin\phi} \diagdown\right]} \tag{4.47}$$

From this pressure distribution one can calculate the aerodynamic center and the slope of the wing lift curve.

D. BASIC LIFT

In the case of basic lift, the pressure distribution is initially prescribed. From this distribution one obtains;

$$\underset{8\times 8}{[\Gamma_\phi]} = \frac{A\!\!R}{8} \underset{8\times 8}{\left[\diagdown c \diagdown\right]} \underset{8\times 8}{[P]} \underset{8\times 8}{\left[\diagdown \sin\phi \diagdown\right]} \tag{4.48}$$

and additionally along the leading edge

$$\underset{8\times 8}{[\Gamma_{\phi\phi L}]} = \frac{A\!\!R}{8} \underset{8\times 8}{\left[\diagdown c \diagdown\right]} \underset{8\times 8}{[P_L]} \tag{4.49}$$

where only the first column of each of the matrices $[\Gamma_{\phi\phi L}]$ and $[P_L]$ is nonzero. Then $[\Gamma_{\phi\phi L}]$ and $[\Gamma_\phi]$ are transposed and substituted into equation 3.30 to get

$$\underset{8\times 8}{[\Gamma]}^T = \underset{8\times 8}{[D_b]}^{-1}\left(\frac{\Delta\phi}{4}\,\underset{8\times 8}{[\Gamma_{\phi\phi L}]}^T + \underset{8\times 8}{[\Gamma_\phi]}^T\right) \tag{4.50}$$

The transpose of $[\Gamma]^T$ is the basic circulation which after being transformed into a 64 by 1 vector is utilized in the following equation to get the wing slope function.

$$\underset{64\times 1}{\{W_p\}} = \underset{64\times 64}{[B]}\,\underset{64\times 1}{\{\Gamma\}} \tag{4.51}$$

The wing slope function can then be used to find the variation in the height of the mean wing surface above the reference xy plane.

V. CONCLUSIONS

From the equations developed and listed in the summary of key relationships of the previous chapter it can be seen that they may be translated into a computer program which calculates the wing slope function and the lift distribution of any designated wing. A further straightforward extension of the analysis (not given in this thesis) then fixes the various aerodynamic parameters of interest such as the slope of the wing lift curve, the location of the aerodynamic center, the pitching moment coefficient about the aerodynamic center, the induced drag and the variation in the height of the mean wing surface above or below the reference xy plane.

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